Low-lying isovector monopole resonances[1]

M. Grigorescu

Abstract: Considering the equality of the proton and neutron Fermi levels as an indication of a phenomenological interaction between pairs of protons and neutrons, low-energy isovector monopole resonances are proved to appear in the superfluid nuclei. The required phenomenological interaction is presented as an isospin symmetry-breaking mean field for the four particle interaction.

1. Introduction

The isovector monopole resonances (IVMR) were predicted in the framework of the nuclear hydrodynamic model [1, 2] as out-of-phase breathing oscillations of the proton and neutron densities. These oscillations represent the isovector counterpart to the isoscalar compression modes, and are generated by the restoring force which appears from the symmetry energy. Experimentally, IVMR were first observed in the charge exchange reactions ^{90}Zr, ^{120}Sn (π^-, π^0) at $T_{\pi^-} = 165$ MeV [3], and were also confirmed recently in the (n, p) reactions on ^{90}Zr [4]. Other experiments [5] have shown their occurrence for a wide range of nuclei, from ^{40}Ca to ^{208}Pb, both normal and superfluid.

The quantum description of the superfluid systems shows a close connection between the velocity potential χ, $(\vec{v} = -\vec{\nabla}\chi)$, describing the flow and the gauge angle φ from the BCS transformation to quasiparticles [2]: $\vec{\nabla}(\chi - \varphi/2m) = 0$, where m is the nucleon mass. For the monopole vibrations, this connection implies a radial dependence of the proton and neutron gauge angles φ_p, φ_n. In contrast to the space dependence, the time dependence of φ_τ, $\tau = p, n$ occurs even if no macroscopic flow is present $(\vec{\nabla}\chi = 0)$. The time-derivative $\dot{\varphi}_\tau/2$ is just the Fermi energy λ_τ [2], and a superfluid system performs in its ground state a free gauge rotation with the angular velocity $\dot{\varphi}_\tau = 2\lambda_\tau$. As is known, the proton and neutron systems are not closed, changing particles through β decay up to the equalisation of the Fermi energies λ_p, λ_n. This can be taken as an indication of a phe-

[1]first printed as preprint FT-353-(1989)/April, by the Institute of Physics and Nuclear Engineering, Bucharest, Romania.

nomenological gauge-restoring interaction, tending to fix not only the space-depending term, but the whole relative angle $\varphi_p - \varphi_n$ at a constant value. Consequently, low-lying IVMR, appearing only in superfluid nuclei are expected, as the isovector counterpart to the isoscalar uniform gauge rotation. The phenomenological character of the gauge-restoring interaction generating these low-lying modes is imposed by its non-commutation with the particle-number operators $\hat{N}_p$, $\hat{N}_n$. So, it can be considered only as a mean-field approximation for a microscopic number-conserving Hamiltonian.

In this paper section 2 presents the phenomenological gauge interaction as a mean field deriving from a microscopic four-particle interaction. Restoring the isospin symmetry by the cranking method, a kinetic energy term contributing to the symmetry energy is obtained. These connections between the gauge-restoring interaction, the four-particle interaction, and the symmetry energy are used at the end of section 2 to estimate the former's strength δ_0. Following the semiclassical treatment of the proton-neutron quadrupole-quadrupole interaction at deformed nuclei [6] in section 3 the effects of the gauge interaction in superfluid nuclei are investigated. This approach gives both the underlying classical picture and the quantised spectrum of the Fermi levels oscillations, allowing for realistic estimates of their energy.

2. The gauge-restoring interaction

The early attempts to construct an isoscalar four-particle interaction were based on the algebraic properties of the pairing operators, of having closed commutation relations with the isospin operators. Denoting by (a,m), $a \equiv (n,l,j)$, the shell-model states (n,l,j,m), the proton-proton, neutron-neutron and the proton-neutron pair creation operators P_+, N_+, R_+, are:

$$P_+ = \frac{1}{2}\sum_{a,m} s_{am} c^\dagger_{pam} c^\dagger_{pa-m} \quad , \quad s_{am} = (-1)^{j-m} \tag{1}$$

$$N_+ = \frac{1}{2}\sum_{a,m} s_{am} c^\dagger_{nam} c^\dagger_{na-m} \tag{2}$$

$$R_+ = \frac{1}{2}\sum_{a,m} s_{am} c^\dagger_{nam} c^\dagger_{pa-m} \quad . \tag{3}$$

If P_-, N_-, R_- are their hermitian conjugates, then

$$[P_+, P_-] = 2P_0 \quad , \quad [N_+, N_-] = 2N_0 \quad , \quad [R_+, R_-] = 2R_0 \tag{4}$$

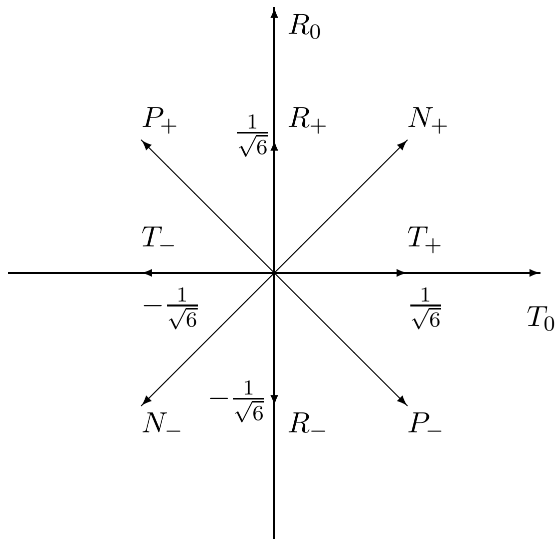


Figure 1. Root diagram of o(5) algebra.

with

$$P_0 = \frac{1}{2}\sum_{a,m}(c^\dagger_{pam}c_{pam} - \frac{1}{2}) \quad (5)$$

$$N_0 = \frac{1}{2}\sum_{a,m}(c^\dagger_{nam}c_{nam} - \frac{1}{2}) \quad (6)$$

$$R_0 = \frac{1}{2}\sum_{a,m}(c^\dagger_{pam}c_{pam} + c^\dagger_{nam}c_{nam} - 1) \quad (7)$$

each couple (P_-, P_0, P_+), (N_-, N_0, N_+), (R_-, R_0, R_+) generating an su(2) algebra. Moreover, if (T_-, T_0, T_+) are the isospin operators:

$$T_+ = \sum_{a,m} c^\dagger_{nam}c_{pam} \quad , \quad T_- = \sum_{a,m} c^\dagger_{pam}c_{nam} \quad (8)$$

$$T_0 = \frac{1}{2}\sum_{a,m}(c^\dagger_{nam}c_{nam} - c^\dagger_{pam}c_{pam}) \quad (9)$$

then $(P_\pm, N_\pm, R_\pm, T_\pm, P_0, N_0)$ generate the second-rank semisimple Lie algebra o(5) [7]. Choosing R_0 and T_0 as basis elements in the Cartan subalgebra, the root diagram can be pictured as in figure 1 [8].

The three commuting operators $\mathcal{P}^\dagger_{-1} = P_+$, $\mathcal{P}^\dagger_0 = R_+/\sqrt{2}$, $\mathcal{P}^\dagger_1 = N_+$ are the components of an isospin vector, and were used in [9, 10] to construct the quadrupling four-particle interaction $H_Q = -G_Q Q^\dagger_0 Q_0/4$, with

$Q_0^\dagger = 2\sum_{\mu=-1}^{1}(-1)^\mu \mathcal{P}_\mu^\dagger \mathcal{P}_{-\mu}^\dagger$. This interaction does not lead to the expected gauge force for the superfluid systems, and consequently in the present work a different choice will be made. Coupling the isovector $\mathcal{P}_\mu^\dagger$ with its Hermitian conjugate $\mathcal{P}_\mu$ to an isospin quadrupole $Q_{2\mu}$, an isoscalar four-particle interaction, separable in the two particles-two holes channel will be defined by

$$H_4 = -G_4 \sum_{\mu=-2}^{2} (-1)^\mu Q_{2\mu} Q_{2-\mu} = -G_4 \sum_{\mu=-2}^{2} Q_{2\mu}^\dagger Q_{2\mu} \quad . \tag{10}$$

With this term, the total microscopic Hamiltonian, including the usual pairing interaction, becomes

$$H = H_0 + H_4 \tag{11}$$

$$H_0 = \sum_{a,m} (\epsilon_{pa} c_{pam}^\dagger c_{pam} + \epsilon_{na} c_{nam}^\dagger c_{nam}) - G_p P_+ P_- - G_n N_+ N_- \quad . \tag{12}$$

The ground state $|g\rangle$ of H includes the 4-particle correlations determined by H_4 and the transition to the gauge-restoring mean field is expected when the sum rule:

$$\sum_{\mu=-2}^{2} \langle g | Q_{2\mu}^\dagger Q_{2\mu} | g \rangle = \sum_{\mu=-2}^{2} \sum_{|n\rangle} |\langle n | Q_{2\mu} | g \rangle|^2 \tag{13}$$

is exhausted by $|g\rangle$:

$$\sum_{\mu=-2}^{2} \langle g | Q_{2\mu}^\dagger Q_{2\mu} | g \rangle \approx \sum_{\mu=-2}^{2} |q_\mu|^2 \quad , \tag{14}$$

$$q_\mu = \langle g | Q_{2\mu} | g \rangle \tag{15}$$

and $q_{\pm 2} \neq 0$. This highly correlated ground state can be approximated by the eigenfunction $|g_\omega\rangle$ of the linearised Hamiltonian H_L:

$$H_L = H_0 - G_4 \sum_{\mu=-2}^{2} (q_\mu Q_\mu^\dagger + q_\mu^* Q_\mu) - \omega T_0 \tag{16}$$

$$H_L |g_\omega\rangle = E_\omega |g_\omega\rangle \tag{17}$$

where q_μ are self-consistently determined by:

$$q_\mu = \langle g_\omega | Q_{2\mu} | g_\omega \rangle \quad . \tag{18}$$

The "cranking" term $-\omega T_0$ was introduced for an approximate projection of the proton and neutron numbers Z, N, because even if H_L commutes with

$\hat{A} = \hat{N}_p + \hat{N}_n$, it does not commute with T_0. So, the parameter ω is fixed by the constraint:

$$\langle g_\omega | T_0 | g_\omega \rangle = \frac{N - Z}{2} \quad . \tag{19}$$

To obtain the function $|g_\omega\rangle$ further approximations will be made, neglecting the proton-neutron pairing interaction and retaining in $|g_\omega\rangle$ only the correlations between pairs of protons and pairs of neutrons. Consequently, H_L becomes:

$$H_L = \sum_{\tau,a,m} \epsilon_{\tau a} c^\dagger_{\tau a m} c_{\tau a m} - \tilde{G}_p P_+ P_- - \tilde{G}_n N_+ N_- + H_g - \omega T_0 \quad , \tag{20}$$

where

$$\tilde{G}_\tau = G_\tau + \frac{2}{\sqrt{6}} G_4 q_0 \tag{21}$$

and $H_g = -2G_4(q_2^* N_+ P_- + q_2 N_- P_+)$, $q_2 = \langle g_\omega | N_+ P_- | g_\omega \rangle$ is the gauge restoring interaction. This Hamiltonian will be restricted to one degenerate $j = 3/2$ level. The interest for this case is justified by the relevance of the O(5) symmetry for the classification of the experimental data. So, the low-lying 0^+ states of the nuclei filling $j = 3/2$ shells, as $1p_{3/2}$, $1d_{3/2}$ and $2p_{3/2}$ can be assigned to the weight vectors of the 14-dimensional irreducible representation of the o(5) algebra. In these multiplets both ground and excited states are included, and figure 2 presents their relative energies with Coulomb corrections[2]. Considering the same pairing constant $\tilde{G}$ and single-particle energy ϵ for both protons and neutrons, the restricted Hamiltonian is

$$H_L = 2\Omega\epsilon + (2\epsilon + \omega) P_0 + (2\epsilon - \omega) N_0 \tag{22}$$
$$-\tilde{G}(P_+ P_- + N_+ N_-) - 2G_4(q_2^* N_+ P_- + q_2 N_- P_+) \quad , \quad \Omega = j + \frac{1}{2} \quad .$$

This Hamiltonian commutes with the operator $\hat{A}$ and with the squares of the proton and neutron quasispin operators $\vec{P}^2$, $\vec{N}^2$, so that its eigenfunctions

[2] The experimental binding and excitation energies have been taken from A. H. Wapstra and K. Bos, *The 1977 atomic mass evaluation*, Atomic Data and Nuclear Data Tables **19** 177 (1977), and P. M. Endt and C. van dèr Leun, *Energy levels of A=21-44 nuclei*, Nucl. Phys. A **214** 1 (1973). When I obtained figure 2 (after the less symmetric diagram for the light nuclei filling the $1p_{3/2}$ shell), I wondered if there was no previous attempt to make such a test of the O(5) symmetry. After a thorough search in literature for several weeks I found a practically unknown paper, by P. Camiz and U. Catani [11], in which the O(5) symmetry was used to classify nuclear states for a wide range of mass numbers. However, I could not find any indication that an effective proton-neutron interaction like H_g of (20) has ever been used before.

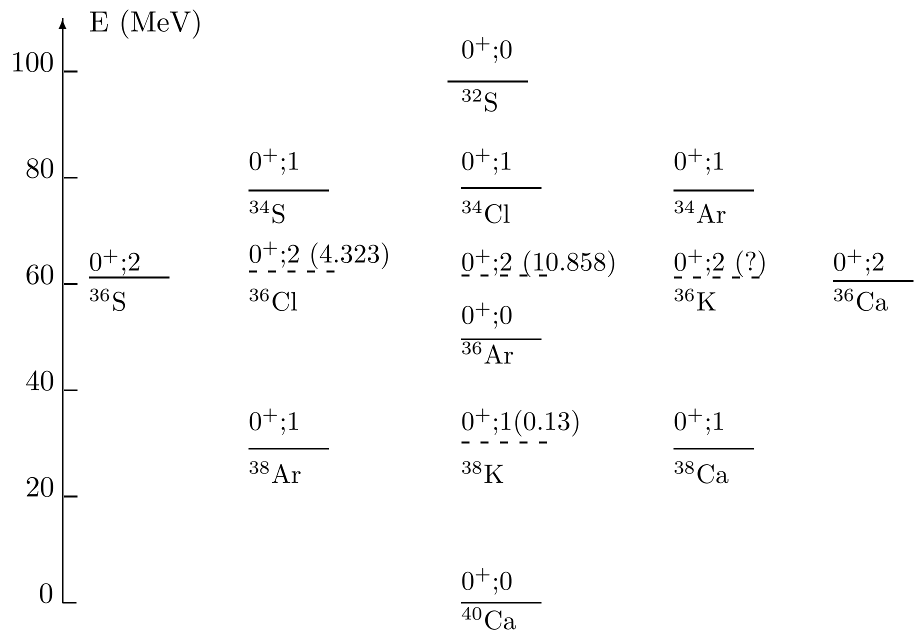


Figure 2. The o(5) multiplet of $J^\pi = 0^+$ ground (solid) and excited (dash) states for the nuclei filling the $1d_{3/2}$ valence level. Whenever available, the excitation energy with respect to the ground state ($(2^+;1)$ for ^{36}Cl, ^{36}K and $(3^+;0)$ for ^{38}K) is given in round brackets (MeV).

are labelled by the eigenvalues E, A_v = number of the valence particles, p and n:

$$
\begin{aligned}
H_L|pnEA_v\rangle_\omega &= E_\omega|pnEA_v\rangle_\omega &(23)\\
\hat{A}|pnEA_v\rangle_\omega &= A_v|pnEA_v\rangle_\omega &(24)\\
\vec{P}^2|pnEA_v\rangle_\omega &= p(p+1)|pnEA_v\rangle_\omega &(25)\\
\vec{N}^2|pnEA_v\rangle_\omega &= n(n+1)|pnEA_v\rangle_\omega \quad . &(26)
\end{aligned}
$$

In addition, $|pnEA_v\rangle_\omega$ depends on ω whose value is fixed by the constraint (19):

$$
{}_\omega\langle pnEA_v|T_0|pnEA_v\rangle_\omega = \frac{N_v - Z_v}{2} \quad . \qquad (27)
$$

So, the function $|pnEA_v\rangle_\omega$ is associated with a given nucleus from the isobaric family determined by A_v. The diagram in figure 2 contains five such families, having in the increasing order of the energies: A_v = 8, 6, 4, 2, 0. From these, the correlation between the proton and neutron pairs is effective

only for $p = n = \Omega/2 = 1$, $A_v = 2\Omega = 4$, and this case will be discussed in the following. Denoting by $|p, p_0\rangle$ and $|n, n_0\rangle$ the seniority eigenfunctions of $\vec{P}^2$, P_0 and $\vec{N}^2$, N_0, respectively, the function $|pnEA_v\rangle_\omega$ can be expressed as:

$$|pnEA_v\rangle_\omega = \sum_{p_0+n_0=A_v/2-\Omega} a_{p_0}|p, p_0\rangle|n, n_0\rangle \quad . \tag{28}$$

When $p = n = 1$, $A_v = 4$, $\Omega = 2$, the coefficients a_{-1}, a_0, a_1, are easily found from the equation $H_L|11E4\rangle_\omega = E_\omega|11E4\rangle_\omega$, and (28) becomes

$$|11E4\rangle_\omega = \frac{1}{\sqrt{1+8\delta_\omega^2\frac{x^2+4\omega^2}{(x^2-4\omega^2)^2}}}\left(\frac{2\delta_\omega}{x+2\omega}|1,-1\rangle|1,1\rangle \right. \tag{29}$$

$$\left. -|1,0\rangle|1,0\rangle + \frac{2\delta_\omega}{x-2\omega}|1,1\rangle|1,-1\rangle\right) \quad .$$

Here $x = 2\tilde{G}+E_\omega$ and $\delta_\omega = 2G_4 q_2$, with q_2 chosen to be real, self-consistently determined by

$$q_2 = \,_\omega\langle 11E4|N_+P_-|11E4\rangle_\omega = \frac{8x\delta_\omega}{(4\omega^2-x^2)(1+8\delta_\omega^2\frac{x^2+4\omega^2}{(x^2-4\omega^2)^2})} \quad . \tag{30}$$

The ω-dependent energy E_ω is analitically obtained from the third-order equation

$$x^3 + 2\tilde{G}x^2 - 4(2\delta_\omega^2 + \omega^2)x - 8\tilde{G}\omega^2 = 0 \quad . \tag{31}$$

Its solutions are

$$E_{g_\omega} = -\frac{8\tilde{G}}{3} - \frac{4}{3}\cos\frac{\phi}{3}\sqrt{\tilde{G}^2 + 6\delta_\omega^2 + 3\omega^2} \quad , \tag{32}$$

$$E_{k_\omega} = -\frac{8\tilde{G}}{3} + \frac{2}{3}(\cos\frac{\phi}{3} + (-1)^k\sqrt{3}\sin\frac{\phi}{3})\sqrt{\tilde{G}^2 + 6\delta_\omega^2 + 3\omega^2} \;,\; k = 1,2 \tag{33}$$

with $\phi \in [0, \pi]$,

$$\tan\phi = \frac{\alpha}{\gamma} \quad , \tag{34}$$

$$\alpha = \sqrt{\frac{64}{27}(\frac{\tilde{G}^2}{3} + 2\delta_\omega^2 + \omega^2)^3 - \gamma^2} \quad , \tag{35}$$

$$\gamma = \frac{8\tilde{G}}{3}(\frac{\tilde{G}^2}{9} + \delta_\omega^2 - \omega^2) \quad , \tag{36}$$

the system having two excited states. At $\omega = 0$, the lower solution E_{g_0}, corresponding to the ground state $|g_0\rangle = |11E_{g_0}4\rangle_0$, is[3]

$$E_{g_0} = -3\tilde{G} - \sqrt{\tilde{G}^2 + 8\delta_0^2} \quad . \tag{37}$$

Using this value, the mean-field parameter $\delta_0 = 2G_4\langle g_0|N_+P_-|g_0\rangle$ can be easily calculated, and one obtains

$$\delta_0 = \sqrt{8G_4^2 - \tilde{G}^2/8} \quad . \tag{38}$$

Differentiating E_ω with respect to ω, the mean value of T_0 can be found

$${}_\omega\langle 11E4|T_0|11E4\rangle_\omega = -\frac{dE_\omega}{d\omega}|_{\delta_\omega = const} \equiv J^\omega \cdot \omega \quad , \tag{39}$$

$$J^\omega = \frac{4(2\tilde{G} + x)^2}{8\tilde{G}\delta_\omega^2 - x(2\tilde{G} + x)^2} \quad . \tag{40}$$

The factor J^ω can be interpreted as the moment of inertia associated with the rotations around the Z axis in isospace. It is not defined as positive, however for the ground state at $\omega = 0$ ($N_v = Z_v$) it proves to be positive:

$$J_g^0 = \frac{32\delta_0^2}{\sqrt{\tilde{G}^2 + 8\delta_0^2}(\sqrt{\tilde{G}^2 + 8\delta_0^2} + \tilde{G})^2} \quad . \tag{41}$$

If $\omega \neq 0$, but small, the energy E_{g_ω} can be approximated as:

$$E_{g_\omega} = E_{g_0} - \frac{J_g^0\omega^2}{2} \quad , \tag{42}$$

and the ground state energy $\mathcal{E}_{g_\omega}$, defined by $\mathcal{E}_{g_\omega} = \langle g_\omega|H|g_\omega\rangle$, becomes within the above approximations

$$\mathcal{E}_{g_\omega} = E_{g_0} + \frac{J_g^0\omega^2}{2} \quad . \tag{43}$$

The parameter ω is calculated from the equations (27), (39), and for small values $(N_v - Z_v)/2$ given to $\langle g_\omega|T_0|g_\omega\rangle$, it will be

$$\omega = \frac{\langle g_\omega|T_0|g_\omega\rangle}{J_g^0} = \frac{N_v - Z_v}{2J_g^0} \quad . \tag{44}$$

[3]For the first excited state Eq. (33) yields $E_{1_0} = -2\tilde{G}$, while the corresponding state vector (29) is $|11E_{1_0}4\rangle_0 = (|1,-1\rangle|1,1\rangle - |1,1\rangle|1,-1\rangle)/\sqrt{2}$.

Replacing ω from (43) by this value, the energies $\mathcal{E}_g(A_v; N, Z)$ of the even-even nuclei from the isobaric family having $A_v = 4$ can be expressed as

$$\mathcal{E}_g(4; N, Z) = E_{g_0} + \frac{(N-Z)^2}{8J_g^0} \quad . \tag{45}$$

Using these results, some numerical estimates of the constants G_4 and δ_0 for the level $1d_{3/2}$ will be made in the following. In this case, the state $|11E_{g_0}4\rangle_0$ having $N_v = Z_v = 2$ corresponds to the isosinglet ground state of ^{36}Ar, while $|11E_g4\rangle_\omega$, $\omega = (N_v - Z_v)/2J_g^0$, for $N_v = 4$, $Z_v = 0$ and $N_v = 0$, $Z_v = 4$ correspond to the $T = 2$ ground states of ^{36}S and ^{36}Ca, respectively. As for the members of the same isomultiplet, the ground state energies $\mathcal{E}_g(4; 4, 0)$ and $\mathcal{E}_g(4; 0, 4)$ of ^{36}S and ^{36}Ca are the same, higher than the ground-state energy $\mathcal{E}_g(4; 0, 0)$ of ^{36}Ar. The difference $\mathcal{E}_g(4; 4, 0) - \mathcal{E}_g(4; 0, 0)$ is just the excitation energy $E_{0^+;2}$ of the state $(J^\pi; T) = (0^+; 2)$ of ^{36}Ar at 10.858 MeV. So, from (45) a first estimate of J_g^0 can be obtained:

$$\frac{1}{J_g^0} = \frac{E_{0^+;2}}{2} \quad . \tag{46}$$

Now, the constant G_4 can be calculated from the second-order equation given by (41). Following [9] in taking $\tilde{G} = 0.6$ MeV, two solutions for G_4 are found: $G_4^{(1)} = 2.56$ MeV, $G_4^{(2)} = 0.08$ MeV. The solution $G_4^{(2)}$ is closer to the value of the quadrupling interaction constant $G_Q = 0.09$ MeV [9], suggesting that $G_4 = G_4^{(2)}$ is the correct choice, if both terms, H_Q and H_4 come out from the same basic four-particle interaction. For this choice the coupling constant of the phenomenological gauge restoring interaction in ^{36}Ar has the value $\delta_0 = 0.07$ MeV.

In the case of other $j = 3/2$ valence levels the numerical values of the parameters $\tilde{G}$, δ_0, must be changed because they are dependent on the mass number A of the $T = 0$ and $T = 2$ isobaric isomultiplets. The average will be used for the pairing strength $\tilde{G} = 21/A$ MeV of the proton and neutron constants $\tilde{G}_p = 23/A$ MeV, $\tilde{G}_n = 19/A$ MeV. The A-dependence of the mean-field parameter δ_0 is unknown, but it will be obtained from the expression of the isorotational energy. The term $(N-Z)^2/8J_g^0$ appearing in (45) represents the contribution of the nucleons lying on the same degenerate $j = 3/2$ level to the symmetry energy from the Weiszäcker mass formula, $W_{sym}(A, N-Z) = k_w(N-Z)^2$, $k_w = 28.1/A$ MeV. This formula describes the average symmetry energy of the nuclei, and in particular, for $N - Z = 4$ it is expected to approximate the energy $E^A_{0^+;2}$ in the light nuclei having $N = Z = A/2$. A comparison with the experimental data shows that

Figure 3. A comparison between the experimental energies $E^A_{0^+;2}$ (* symbols), and $W_{sym}(A,4)$ (solid line).

$W_{sym}(A,4)$ is systematically greater than $E^A_{0^+;2}$ (figure 3), but except for the mass region $A \leq 28$ the relative difference $W_{sym}(A,4)/E^A_{0^+;2} - 1$ decreases with A, becoming 1.5 % for ^{52}Fe. Consequently, for $A \geq 28$, $W_{sym}(A,4)$ reproduces quite well the value of $E^A_{0^+;2}$, proving in particular that J^0_g is proportional to A. As $\tilde{G} \sim A^{-1}$, the formula (41) also implies an average A^{-1} dependence for δ_0, which in the range $A \geq 28$ is determined by its value at $A = 36$ to be $\delta_0 = 2.7/A$ MeV.

3. The Fermi levels oscillations in superfluid nuclei

The ground state of the superfluid systems contains correlations between the same kind of particles, and is well approximated by the BCS function. For a single proton level the pairing Hamiltonian H_p and the BCS function become:

$$H_p = \epsilon \hat{N}_p - \tilde{G} P_+ P_- \tag{47}$$

$$|BCS\rangle_{(\varphi,\lambda)} = e^{2zP_+ - 2z^*P_-}|0\rangle = e^{-i\varphi \hat{N}_p/2}|BCS\rangle_{(0,\lambda)} \tag{48}$$

where λ is the Fermi energy and $z = \rho e^{-i\varphi}$. Explicitly, $|BCS\rangle_{(0,\lambda)}$ is determined as the ground-state eigenfunction for the Hamiltonian $H' =$

$H_{pL}-\lambda\hat{N}_p$, containing the linearised Hamiltonian H_{pL}:

$$H_{pL} = \epsilon\hat{N}_p - \Delta(P_+ + P_-) \quad , \tag{49}$$

$$\Delta = \tilde{G}\langle BCS_{(0,\lambda)}|P_+|BCS_{(0,\lambda)}\rangle = \frac{\tilde{G}\Omega}{2}\sin 4\rho \quad . \tag{50}$$

The term $-\lambda\hat{N}_p$ appearing here can be interpreted as a cranking term restoring the symmetry broken by H_{pL}. To reach this interpretation, one starts from the time-dependent Schrödinger equation $i\partial_t|\Psi(t)\rangle = H_{pL}|\Psi(t)\rangle$ written for the Hamiltonian $H_{pL}(t) = e^{-i\varphi(t)\hat{N}_p/2}H_{pL}e^{i\varphi(t)\hat{N}_p/2}$, generated by a uniform gauge rotation with the angular velocity $\dot{\varphi} = 2\lambda$ from H_{pL}. Its solution $|\Psi(t)\rangle = e^{-i\varphi(t)\hat{N}_p/2}|\Psi_0(t)\rangle$, $|\Psi_0(t)\rangle = e^{-iE'_g t}|BCS\rangle_{(0,\lambda)}$ contains the previous eigenfunction of H', $|BCS\rangle_{(0,\lambda)}$, whose eigenvalue E'_g and parameter ρ are determined by

$$H'|BCS\rangle_{(0,\lambda)} = E'_g|BCS\rangle_{(0,\lambda)} \tag{51}$$

to be

$$E'_g = -E\Omega\left(1-\frac{\epsilon-\lambda}{E}\right) \quad , \quad E = \sqrt{(\epsilon-\lambda)^2+\Delta^2} = \frac{\tilde{G}\Omega}{2} \quad , \tag{52}$$

$$\cos 4\rho = 2\frac{\epsilon-\lambda}{\tilde{G}\Omega} \quad . \tag{53}$$

Also, the energy function $\mathcal{E}$ and the mean number of particles $\mathcal{N}$ of the system are:

$$\mathcal{E} = \langle BCS_{(0,\lambda)}|H_p|BCS_{(0,\lambda)}\rangle = \epsilon\Omega - \frac{\epsilon^2 I_p}{2} - \frac{\Omega^2}{2I_p} + \frac{I_p\lambda^2}{2} + \mathcal{O}(\mathcal{N}/\Omega), \tag{54}$$

$$\mathcal{N} = \langle BCS_{(0,\lambda)}|\hat{N}_p|BCS_{(0,\lambda)}\rangle = \Omega - \epsilon I_p + \lambda I_p \quad , \tag{55}$$

where $I_p = 2/\tilde{G}$ is the "moment of inertia" for the rotation generated by $\hat{N}_p$, and $\mathcal{O}(\mathcal{N}/\Omega) = -(\epsilon-\lambda-\tilde{G}\Omega/2)^2/\tilde{G}\Omega$ is usually neglected, assuming Ω large. A direct calculation shows that similar formulae are obtained for the case of 2Ω levels distributed with a constant density $\bar{\rho} = 2\Omega/(\epsilon_2-\epsilon_1)$ between the energies $\epsilon_2 > \epsilon_1$:

$$\mathcal{E}^c = \bar{\epsilon}\Omega - \frac{\bar{\epsilon}^2 I_p^c}{2} - \frac{\Omega^2}{2I_p^c} + \frac{I_p^c\lambda^2}{2} \quad , \quad \bar{\epsilon} = \frac{\epsilon_1+\epsilon_2}{2} \quad , \tag{56}$$

$$\mathcal{N}^c = \Omega - \bar{\epsilon}I_p^c + \lambda I_p^c \quad , \tag{57}$$

except for the moment of inertia which becomes $I_p^c = \bar{\rho}/\coth(2/\bar{\rho}\tilde{G})$ [12].

To investigate the effects of the interaction $-\delta_0(N_+P_- + N_-P_+)$ for a superfluid system, the microscopic Hamiltonian H_L of (20) will be treated semiclassically[4], using the variational method for the time-dependent trial function $|\Psi\rangle$. If $\{a_i, i = 1, n\}$ are the real time-dependent parameters of $|\Psi\rangle$, then the variational equations:

$$\delta_{a_i} \int dt \langle\Psi|i\partial_t - H_L|\Psi\rangle = 0 \tag{58}$$

give $|\Psi\rangle$ as an approximation of the exact solution of the equation $i\partial_t|\Psi(t)\rangle = H_L|\Psi\rangle$. Explicitly (58) becomes:

$$\sum_{j=1}^{n} B_{ij}\dot{a}_j = \frac{\partial \mathcal{H}}{\partial a_i} \quad , \quad \mathcal{H} = \langle\Psi|H_L|\Psi\rangle \quad , \tag{59}$$

$$B_{ij} = 2\mathrm{Im}\langle\partial_j\Psi|\partial_i\Psi\rangle \quad , \quad \partial_j \equiv \partial/\partial a_j \quad , \tag{60}$$

describing a classical dynamical system on the trial functions manifold [13]. Particularly, choosing $|\Psi\rangle$ as a product of two BCS functions, one for protons and another for neutrons, $|\Psi\rangle = |BCS_p\rangle|BCS_n\rangle$, this dynamical system becomes Hamiltonian. For the one-level case,

$$|\Psi\rangle = e^{2z_pP_+ - 2z_p^*P_-}e^{2z_nN_+ - 2z_n^*N_-}|0\rangle \quad , \quad z_\tau = \rho_\tau e^{-i\varphi_\tau} \quad , \tau = p, n$$

and the gauge angles together with the mean number of particles $\mathcal{N}_\tau = \langle\Psi|\hat{N}_\tau|\Psi\rangle = 2\Omega\sin^2 2\rho_\tau$, give the canonical coordinates $\{Q_\tau, P_\tau\}_{\tau=p,n}$:

$$Q_\tau = \varphi_\tau \quad , \quad P_\tau = \frac{\mathcal{N}_\tau}{2} \quad . \tag{61}$$

Their time evolution is determined by the Hamilton equations:

$$\dot{Q}_\tau = \frac{\partial \mathcal{H}}{\partial P_\tau} \quad , \quad \dot{P}_\tau = -\frac{\partial \mathcal{H}}{\partial Q_\tau} \quad , \tag{62}$$

with

$$\mathcal{H} = 2(\epsilon - \frac{\tilde{G}\Omega}{2})(P_p + P_n) + \tilde{G}(1 - \frac{1}{\Omega})(P_p^2 + P_n^2) \tag{63}$$
$$-2\delta_0\sqrt{P_pP_n(\Omega - P_p)(\Omega - P_n)}\cos(\varphi_p - \varphi_n)$$

[4] This method is expected to yield the average properties of the collective excitation produced by the interaction term $-\delta_0(N_+P_- + N_-P_+)$ when Ω is large, becoming increasingly accurate when $\delta_0/\tilde{G}$ increases (M. Grigorescu, Can. J. Phys. **78** 119 (2000)).

or equivalently, by the solutions $\{\rho_\tau(t), \varphi_\tau(t)\}_{\tau=p,n}$ of the system

$$\begin{aligned}
\dot\rho_p &= -\frac{\delta_0\Omega}{4}\sin 4\rho_n \sin(\varphi_p-\varphi_n) \\
\dot\rho_n &= \frac{\delta_0\Omega}{4}\sin 4\rho_p \sin(\varphi_p-\varphi_n) \\
\dot\varphi_p &= 2\epsilon - \tilde G - \tilde G(\Omega-1)\cos 4\rho_p - \delta_0\Omega\cot 4\rho_p \sin 4\rho_n \cos(\varphi_p-\varphi_n) \\
\dot\varphi_n &= 2\epsilon - \tilde G - \tilde G(\Omega-1)\cos 4\rho_n - \delta_0\Omega\cot 4\rho_n \sin 4\rho_p \cos(\varphi_p-\varphi_n) \quad .
\end{aligned} \tag{64}$$

If $\delta_0 = 0$, then $\dot\rho_p = 0$, $\dot\rho_n = 0$, and the number of particles is fixed by the initial values $\rho_p(0)$, $\rho_n(0)$, without additional constraints to the variational equation (58). When $\delta_0 \neq 0$ it is convenient to use the "centre of mass" and "relative" canonical coordinates (Q, P) and $(\Phi, \wp)$, respectively, defined by:

$$Q = \frac{Q_p+Q_n}{2} \quad , \quad P = P_p + P_n \tag{65}$$

$$\Phi = Q_p - Q_n \quad , \quad \wp = \frac{P_p - P_n}{2} \quad . \tag{66}$$

In these coordinates $\mathcal{H}$ becomes

$$\mathcal{H}(P,\wp,\Phi) = 2(\epsilon - \frac{\tilde G\Omega}{2})P + \tilde G(1-\frac{1}{\Omega})(\frac{P^2}{2}+2\wp^2) \tag{67}$$

$$-2\delta_0\sqrt{(\frac{P^2}{4}-\wp^2)[(\Omega-\frac{P}{2})^2-\wp^2]}\cos\Phi$$

It is appearing clearly that $P = A/2$, $A = \mathcal{N}_p + \mathcal{N}_n$, is time-independent. Moreover, the set $M = \{(Q,P,\Phi,\wp), \Phi = 0, \wp = 0\}$ consists of closed orbits, distinguished by A. For each $A = 0, 4, 8, ..., 4\Omega$ there is an orbit in M having $\Phi(t) = 0$, $\wp(t) = 0$, $P(t) = A/2$, and $Q(t) = Q_0 + 2\lambda t$, with $2\lambda = 2[\epsilon - (\tilde G + \delta_0)\Omega/2] + [\tilde G(1-\Omega^{-1}) + \delta_0]A/2$. If $(\Phi, \wp)$ are not zero, but small, the orbits can be found by expanding $\mathcal{H}(P,\wp,\Phi)$ around the point $(P, 0, 0)$. Within this approximation $\mathcal{H}$ becomes:

$$\mathcal{H} = 2(\epsilon - \frac{\tilde G + \delta_0}{2}\Omega)P + [\tilde G(1-\frac{1}{\Omega}) + \delta_0]\frac{P^2}{2} + k\wp^2 + \frac{C\Phi^2}{2} \quad , \tag{68}$$

$$k = 2\tilde G(1-\frac{1}{\Omega}) - 2\delta_0 - \frac{16\delta_0\Omega^2}{A(A-4\Omega)} \quad , \quad C = 2\delta_0(\frac{\Delta}{\tilde G})^2 \quad , \tag{69}$$

and the equations describing the time evolution of the relative coordinates $(\Phi, \wp)$ can be easily integrated to:

$$\Phi(t) = \Phi_0 \sin\omega_v t \quad , \quad \wp(t) = \wp_0 \cos\omega_v t \quad , \tag{70}$$

$$\wp_0 = \frac{\omega_v}{2k}\Phi_0 \quad , \quad \omega_v = \sqrt{2kC} \quad . \tag{71}$$

The interpretation of the derivatives $\dot{Q}_p/2$, $\dot{Q}_n/2$ as Fermi energies associates the solution given above for the $N = Z$ ($\wp = 0$) nuclei, to small oscillations in opposite directions of the proton and neutron Fermi levels. Pictorially, in the Dasso-Vitturi representation [14], they correspond to relative angular oscillations of the proton and neutron deformed densities in gauge space. Such oscillations, having an isovector character, are allowed by the diffuseness of the Fermi surface for the superfluid systems. Their quantisation [15] leads to a harmonic oscillator spectrum, whose first excited state has the energy ω_v[5]. For numerical estimates of this energy it is convenient to replace the constant k obtained above for a single level by the realistic value $k = 16k_w = 449.6/A$ MeV, as given by the symmetry energy. The constant C will be calculated for $N = Z$ nuclei with half-filled valence shells ($\sin 4\rho = 1$), taking $\delta_0 = 2.7/A$ MeV as before, and $\Delta = \tilde{G}\Omega/2$. Here $\tilde{G} = 21/A$ MeV, and $\Omega \approx 0.5(3A/2)^{2/3}$ is half the Fermi level degeneracy for the harmonic oscillator potential [2]. Using these constants, the energy of the isovector monopole vibration becomes $\omega_v = 22.8A^{-1/3}$ MeV.

4. Conclusions

The gauge-restoring interaction $-\delta_0(N_+P_- + N_-P_+)$ introduced in order to fix the same proton and neutron Fermi levels in superfluid nuclei was proved to be strongly connected with the symmetry energy and with the four-particle interaction. For the non-superfluid nuclei, its non-commutation with T_0 leads to a kinetic energy term corresponding to collective rotations around the Z-axis in isospace (isorotations). In contrast to the purely kinematic symmetry energy determined by the Pauli principle in the Fermi gas model [16], the isorotational symmetry energy occurs dynamically from the proton-neutron correlations. This result is supported by the experimental data on light nuclei, which show large mass differences $E_{0^+;2}$ between the even-even members of the $T = 0$ and $T = 2$ isomultiplets having the valence nucleons on the same degenerate level. An energy $E_{0^+;2}$ close to the value predicted by the symmetry energy term from the Weizsäcker mass formula in its simplest form, was taken as an indication of the isorotational origin of the $T = 2$ multiplet. This was the case for the even-even isobars with $A \geq 28$, and the energy $E_{0^+;2}$ corresponding to $A = 36$ was used to estimate the constant δ_0 as $\delta_0 = 2.7/A$ MeV. Because the microscopic proton-neutron

[5]When $A = 4$, $\Omega = 2$ this vibration mode correponds to the first excited state $|11E_14\rangle_0$, with the excitation energy $e_x = \tilde{G} + (\tilde{G}^2 + 8\delta_0^2)^{1/2}$. The difference $e_x - \omega_v$ is positive but decreases with δ_0, and for $\delta_0 > 1.5\tilde{G}$ it falls below 10 % of e_x.

interaction is number conserving, δ_0 was considered to be a mean-field parameter. Such a mean field corresponds to the four-particle interaction expressed in (10), whose strength G_4 gives δ_0 self-consistently. Estimating G_4 for ^{36}Ar, a value close to the quadrupling constant G_Q previously used in [9] was obtained. This result shows that the isospin quadrupole interaction (10) may represent an alternative to quadrupling for the charge-independent treatment of the nuclear correlations.

In the superfluid nuclei the gauge-restoring interaction leads to isovector oscillations of the Fermi levels, superposed over uniform isorotations. Their energy $\omega_v = 22.8A^{-1/3}$ MeV was estimated for $A \geq 28$ nuclei with half-filled valence shells, considering the same constant δ_0 as for the $j = 3/2$ shell. Due to the low value of ω_v the Fermi levels oscillations are different from the $170A^{-1/3}$ MeV resonances presented in [3, 4, 5], but can appear as low-lying IVMR in high resolution charge exchange or electron scattering experiments. Their occurrence is expected for the superfluid nuclei on the β stability line, the further experimental investigation representing a test for the gauge angles dynamics presented above.

References

[1] J. Damgaard, V. V. Gortchakov, G. M. Vagradov and A. Molinari, Nucl. Phys. A **121** 625 (1968).

[2] A. Bohr and B. R. Mottelson, *Nuclear Structure* Vol. II, (New York: Benjamin) 1975.

[3] J. D. Bowman et. al., Phys. Rev. Lett. **50**, 1195 (1983).

[4] T. D. Ford et. al., Phys. Lett. B **195**, 311 (1987).

[5] A. Erell et al., Phys. Rev. Lett. **52**, 2134 (1984).

[6] M. Grigorescu, IPNE preprint FT-326-1988.[6]

[7] K. T. Hecht, Nucl. Phys. A **102**, 11 (1967).

[8] R. E. Behrends, J. Dreitlein, C. Fronsdal and W. Lee, Rev. Mod. Phys. **34**, 1 (1962).

[9] J. Eichler and M. Yamamura, Nucl. Phys. A **182**, 33 (1972).

[6] nucl-th/0306052

[10] M. Apostol, I. Bulboacă, F. Cârstoiu, O. Dumitrescu and M. Horoi, Nucl. Phys. A **470**, 64 (1987).

[11] P. Camiz and U. Catani, Nuovo Cimento B **64**, 56 (1969).

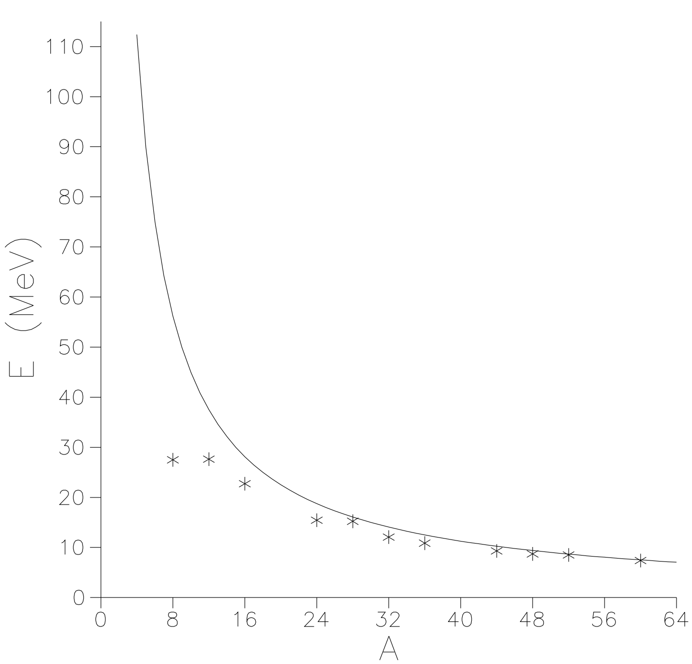
[12] P. Ring and P. Schuck, *The Nuclear Many-Body Problem*, (Berlin: Springer) 1980, p. 240.

[13] K. K. Kan, Phys. Rev. C **24**, 279 (1981).

[14] C. H. Dasso and A. Vitturi, Phys. Rev. Lett. **59**, 634 (1987).

[15] V. Ceauşescu and A. Gheorghe, "Classical Limit and Quantization of Hamiltonian Systems" in Proc. Int. School *Symmetries and Semiclassical Features of Nuclear Dynamics*, Poiana Braşov, România, edited by A. A. Răduţă, (Berlin: Springer) 1987, p. 69.

[16] A. de Shalit and H. Feshbach, *Theoretical Nuclear Physics* Vol. I, (New York: Wiley) 1974, p. 127.


E (MeV)
0
10
20
30
40
50
60
70
80
90
100
110
0
8
16
24
32
40
48
56
64
A